Wrocław University of Science and Technology
Doctoral School

COURSE CARD

1. Basic information

Course name in English:	Dynamical Systems and Ergodic Theory	
Course name in Polish:	Układy Dynamiczne i Teoria Ergodyczna	
Number of hours:	Select number of hours	
Type of course:	Elective course	
Form of course:	lecture	
Code of course:		
Course leader:	Tomasz Downarowicz	
Faculty of the course leader:	W13 Faculty of Pure and Applied Mathematics	
Email address of the course leader:	downar@pwr.edu.pl	
Scientific discipline(s) assigned to the course (doctoral students representing the marked disciplines can participate in the course):	Architecture and urban planning	☐
	Automation, electronic, electrical engineering and space technologies	☐
	Information and communication technology	☐
	Biomedical engineering	☐
	Chemical engineering	☐
	Civil engineering, geodesy and transport	☐
	Materials engineering	☐
	Mechanical engineering	☐
	Environmental engineering, mining, and energy	☐
	Mathematics	☒
	Chemical sciences	☐
	Physical sciences	☐
	Management and quality studies	☐

2. Objectives

C1 Students will learn selected notions of the Theory of Dynamical Systems including Ergodic Theory, and main theorems concerning these notions.
C2 The student will recognize the connections between the notions of the Theory of Dynamical Systems
and notions from other areas of mathematics, such as Stochastic Processes, Differential Equations, or Number Theory.
C3 The student should acquire the skills of applying selected tools of the Theory of Dynamical Systems
to solving problems, including problems from other areas of mathematics and physics.

3. Content

Detailed information about the course content, including topics and form of classes.

No.	Topic	Number of hours	Form of classes
1	Abstract notion of a dynamical system with the action of a group or subgroup, examples	2	Select form
2	Topological dynamical systems, transitivity, minimality, recurrence, factors, conjugacy, proximal pairs, asymptotic pairs, Li-Yorke chaos	2	Select form
3	Symbolic systems. Properties, characterization, applications	2	Select form
4	Connections with Number Theory – IP sets and other. Mention of Sarnak Conjecture	2	Select form
5	Measure-preserving transformations. Factors and isomorphism. Recalling the basic theorems: Poincare recurrence, Rokhlin Lemma and Ergodic Birkhoff Theorem. Connections with stochastic processes.	2	Select form
6	Invariant measures in a topological system. Simplex of measures. Mention of the Furstenberg Conjecture.	2	Select form
7	Spectral properties of a dynamical system. Koopman Operator and Markov Operator. Spectral Theorem. Point Spectrum, pure point spectrum (measure theoretic and topological).	2	Select form
8	Equicontinuity. Maximal equicontinuous factor, Kronecker factor. Group rotation. Halmos – von Neumann Theorem.	2	Select form
9	Mixing, weak mixing, Bernoulli systems.	2	Select form
10-12	Various constructions of dynamical systems: induced maps, suspensions, skew products, group extensions, semicocycle extensions, special flows, elements of the theory of Smooth Systems	6	Select form
13	Topological and measurable joinings. Disjointness	2	Select form
14-15	Topological and measure-theoretic entropy. Conditional entropy.	4	Select form

4. Prerequisites

List of prerequisites relating to knowledge, skills and other competences for course participants.

Basic Measure Theory, Basic Functional Analysis, Basic Topology

5. Learning outcomes

List of learning outcomes at level 8 of the Polish Qualifications Framework assigned to the course (mark the learning outcomes in the last column).

Symbol	Learning outcome	
	KNOWLEDGE. Doctoral student knows and understands:	

SzD_W3	the main trends in the development of the scientific or artistic disciplines covered in the curricula;	☒
SzD_W4	research methodology;	☐
SzD_W5	the rules for the dissemination of scientific results, including in open access mode;	☐
SzD_W6	the fundamental dilemmas of modern civilization;	☐
SzD_W7	the legal and ethical conditions of scientific activity;	☐
SzD_W8	the economic and other relevant conditions of scientific activity;	☐
SzD_W9	basic principles of knowledge transfer to the economic and social spheres and commercialisation of results of scientific activity and know-how related to these results.	☐
	SKILLS. Doctoral student is able to:	
SzD_U2	use knowledge from different fields of science or art to creatively identify, formulate and innovatively solve complex problems or perform research tasks, in particular: - define the purpose and subject of scientific research, formulate a research hypothesis, - develop research methods, techniques and tools, and use them creatively, - draw conclusions on the basis of scientific research; critically analyse and evaluate the results of scientific research, expertise and other creative work and their contribution to knowledge development; transfer the results of scientific activities to the economic and social spheres;	☒
SzD_U3	communicate on specialised topics to the extent that they enable an active participation in the international scientific community;	☐
SzD_U4	disseminate research results, including in popular forms;	☐
SzD_U5	initiate debates and participate in a scientific discourse;	☐
SzD_U6	be able to speak a foreign language at B2 level of the Common European Framework of Reference for Languages to a level that enables them to participate in the international scientific and professional environment;	☐
SzD_U7	plan and implement an individual or collective research or creative activity, including in an international environment;	☐
SzD_U8	independently plan and act for one's own development and inspire and organize the development of others;	☐
SzD_U9	plan classes or groups of classes and implement them using modern methods and tools.	☐
	SOCIAL COMPETENCES. Doctoral student is ready to:	
SzD_K3	fulfilling the social obligations of researchers and creators, initiate public interest activities, thinking and acting in an entrepreneurial way;	☐
SzD_K4	maintaining and developing the ethos of research and creative environments, including: - carrying out scientific activities in an independent manner, - respecting the principle of public ownership of research results, taking into account the principles of intellectual property protection.	☒

6. Evaluation

Short description of the method(s) used to evaluate the learning outcomes assigned to the course, e.g., exam, test, report, presentation, etc.

test

7. Teaching methods

Short description of the teaching methods used during the course, e.g., multimedia presentation, discussion, literature studies, developing written documents, own work, etc.

N1 lecture
N2 consultations
N3 written assignments: problem solutions

8. Literature

List of primary and secondary literature used to prepare the course and including additional knowledge for participants, e.g., books, textbooks, research papers, standards, web pages, etc.

PRIMARY LITERATURE:
[1] Peter Walters: An Introduction to Ergodic Theory, Springer 1982
[2] Karl Petersen: Ergodic Theory
[3] J. de Vries: Elements of Topological Dynamics
SECONDARY LITERATURE:
[4] T. Downarowicz: Entropy in Dynamical Systems , Cambridge University Press 2011

9. Other remarks

Additional remarks, comments, (e.g., language of the course)

In Polish or English