Wrocław University of Science and Technology
Doctoral School

COURSE CARD

1. Basic information

Course name in English:	Piecewise-smooth Dynamical Systems – Bifurcation Analysis and Numerical Simulations	
Course name in Polish:	Układy Dynamiczne Kawałkami Gładkie – Analiza Bifurkacji i Metody Numeryczne	
Number of hours:	30	
Type of course:	Elective course	
Form of course:	mixed forms (combination of lecture, seminar and laboratory)	
Code of course:		
Course leader:	*Dr hab. inż. Piotr Kowalczyk*	
Faculty of the course leader:	W13 Faculty of Pure and Applied Mathematics	
Email address of the course leader:	Piotr.s.kowalczyk@pwr.edu.pl	
Scientific discipline(s) assigned to the course (doctoral students representing the marked disciplines can participate in the course):	Architecture and urban planning	☐
	Automation, electronic, electrical engineering and space technologies	☒
	Information and communication technology	☐
	Biomedical engineering	☐
	Chemical engineering	☐
	Civil engineering, geodesy and transport	☐
	Materials engineering	☐
	Mechanical engineering	☒
	Environmental engineering, mining, and energy	☐
	Mathematics	☒
	Chemical sciences	☒
	Physical sciences	☒
	Management and quality studies	☐

2. Objectives

C1. The student will learn techniques and concepts which allow to perform local bifurcation analysis in piecewise smooth dynamical systems, and in particular in systems with discontinuous vector fields and in vector fields combined with reset maps.

C2. The student will learn how to investigate bifurcations in piecewise-smooth systems and will learn how to implement analytical results to perform numerical analysis of bifurcations in piecewise-smooth systems. In particular, the student will learn how to detect invariant sets such as equilibrium points and limit cycles and detect nonsmooth bifurcations of these sets, as well as perform so-called parameter continuation of invariant sets and bifurcation curves.

C3. The student will be able to apply piecewise-smooth framework to describe problems of relevance to applications, for example, in control theory and biomechanics. The student will be able to analyse the derived mathematical models from the standpoint of local bifurcation theory, that is, investigate the existence and stability of invariant sets under parameter variations.

3. Content

Detailed information about the course content, including topics and form of classes.

No.	Topic	Number of hours	Form of classes
1	Examples of the application of piecewise-smooth dynamical systems in control engineering, mechanics, biomechanics and neurosciences. Introduction.	2	lecture
2	Phase space of Filippov systems. The definition of sliding vector field on the discontinuity set (switching manifold).	2	lecture
3	Bifurcation analysis of equilibrium points in Filippov systems.	2	lecture
4	Poincare section and analysis of the stability of limit cycles in piecewise-smooth dynamical systems. Introduction of the concepts of monodromy matrix and discontinuity mapping.	2	lecture
5	Bifurcations of limit cycle in Filippov systems. Normal form derivation for grazing-sliding bifurcation.	2	lecture
6	Analysis of two-parameter discontinuity induced bifurcations of limit cycles in Filippov systems. An introduction to classification.	2	lecture
7	Piecewise-smooth systems with state resets; canonical examples.	2	lecture
8	Normal form derivation for grazing bifurcation in systems with state resets.	2	lecture
9	Analysis of the dynamics and an unfolding of a DIB in a chosen piecewise-smooth system.	2	seminar
10	Analysis of two-parameter discontinuity bifurcation of limit cycles. Unfolding of two-parameter DIBs in a chosen piecewise-smooth system.	2	seminar
11	Methods for numerical explorations of the dynamics of piecewise-smooth systems. Direct integration methods (lab. sessions). Programming in Matlab environment.	4	laboratory
12	Parameter continuation of invariant sets in piecewise-smooth dynamical systems (lab. sessions). Programming in Matlab environment.	6	laboratory

4. Prerequisites

List of prerequisites relating to knowledge, skills and other competences for course participants.

1. Knowledge of Calculus and Linear Algebra. Some knowledge of asymptotic methods will be useful. Knowledge of bifurcation analysis and numerical methods for the parameter continuation of invariant sets in smooth dynamical systems will be also useful. It is expected that the students will be able to implement numerical methods in Matlab environment, or by means of other programming languages such as, for instance Pyhton, depending on the abilities and preferences of a student.
2. Skills to be able to expand knowledge individually and access appropriate scientific resources.

5. Learning outcomes

List of learning outcomes at level 8 of the Polish Qualifications Framework assigned to the course (mark the learning outcomes in the last column).

Symbol	Learning outcome	
	KNOWLEDGE. Doctoral student knows and understands:	
SzD_W3	the main trends in the development of the scientific or artistic disciplines covered in the curricula;	☒
SzD_W4	research methodology;	☒
SzD_W5	the rules for the dissemination of scientific results, including in open access mode;	☐
SzD_W6	the fundamental dilemmas of modern civilization;	☐
SzD_W7	the legal and ethical conditions of scientific activity;	☐
SzD_W8	the economic and other relevant conditions of scientific activity;	☐
SzD_W9	basic principles of knowledge transfer to the economic and social spheres and commercialisation of results of scientific activity and know-how related to these results.	☐
	SKILLS. Doctoral student is able to:	
SzD_U2	use knowledge from different fields of science or art to creatively identify, formulate and innovatively solve complex problems or perform research tasks, in particular: - define the purpose and subject of scientific research, formulate a research hypothesis, - develop research methods, techniques and tools, and use them creatively, - draw conclusions on the basis of scientific research; critically analyse and evaluate the results of scientific research, expertise and other creative work and their contribution to knowledge development; transfer the results of scientific activities to the economic and social spheres;	☒
SzD_U3	communicate on specialised topics to the extent that they enable an active participation in the international scientific community;	☒
SzD_U4	disseminate research results, including in popular forms;	☐
SzD_U5	initiate debates and participate in a scientific discourse;	☐
SzD_U6	be able to speak a foreign language at B2 level of the Common European Framework of Reference for Languages to a level that enables them to participate in the international scientific and professional environment;	☒

SzD_U7	plan and implement an individual or collective research or creative activity, including in an international environment;	☒
SzD_U8	independently plan and act for one's own development and inspire and organize the development of others;	☐
SzD_U9	plan classes or groups of classes and implement them using modern methods and tools.	☐
	SOCIAL COMPETENCES. Doctoral student is ready to:	
SzD_K3	fulfilling the social obligations of researchers and creators, initiate public interest activities, thinking and acting in an entrepreneurial way;	☐
SzD_K4	maintaining and developing the ethos of research and creative environments, including: - carrying out scientific activities in an independent manner, - respecting the principle of public ownership of research results, taking into account the principles of intellectual property protection.	☒

6. Evaluation

Short description of the method(s) used to evaluate the learning outcomes assigned to the course, e.g., exam, test, report, presentation, etc.

The evaluation of learning outcomes and final mark for the course will be based on a written report and presentations on a chosen problem. See teaching methods for further details.

7. Teaching methods

Short description of the teaching methods used during the course, e.g., multimedia presentation, discussion, literature studies, developing written documents, own work, etc.

Lecture – standard form

Seminar – in the seminar sessions the students will work in groups (2 or 3 students) on mathematical modeling and analysis of chosen problems of relevance to applications using piecewise smooth dynamical systems farmework. Each student will make a presenaton and write a report on his/her results.

Laboratory session will be complementary to the seminar sessions; chosen mathematical models will be analysed numerically by means of methods which apply to piecewise smooth systems. Each student will make a presenaton and write a report on his/her results.

Consultations

8. Literature

List of primary and secondary literature used to prepare the course and including additional knowledge for participants, e.g., books, textbooks, research papers, standards, web pages, etc.

PRIMARY LITERATURE:

[1] M. di Bernardo, C. Budd, A. R. Champneys, *P. Kowalczyk, Piecewise-smooth Dynamical Systems: Theory and Applications*, Springer, 2008

[2] Yu. A. Kuznetsov, S. Rinaldi, and A. Gragnani, *One-Parameter Bifurcations in Planar Filippov Systems,* Int. J. Bifurcation Chaos **13**, 2157, 2003

[3] R. Seydel, *From Equilibrium to Chaos: Practical Bifurcation and Stability Analysis*, Elsevier, 1989

SECONDARY LITERATURE:

[1] A. B. Nordmark and *P. Kowalczyk*, *A codimension-two scenario of sliding solutions in grazing-sliding bifurcations*, Nonlinearity **19(1)** pp. 1-26, 2006

[2] P. *Kowalczyk*, M. di Bernardo, A. R. Champneys, S. J. Hogan, M. Homer, Yu. A. Kuznetsov, A. B. Nordmark and P. T. Piiroinen, *Two-parameter nonsmooth bifurcations of limit cycles: classification and open problems,* International Journal of bifurcation and chaos, **Vol. 16 No. 3,** pp. 601-629, 2006

[3] P. T. Piiroinen and Yu. A. Kuznetsov *An Event-Driven Method to Simulate Filippov Systems with Accurate Computing of Sliding Motions* in ACM Transactions on Mathematical Software 34(3), May 2008

[4] F. Dercole and Yu. A. Kuznetsov, SlideCont: An Auto97 Driver for Bifurcation

Analysis of Filippov Systems

[5] H. Dankowicz and F. Schilder, *Recipes for Continuation*, SIAM 2013

9. Other remarks

Additional remarks, comments, (e.g., language of the course)

English